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Effects of Self-Correlation Time and Cross-Correlation Time of Additive and Multiplicative Colored Noises for Dynamical Properties of a Bistable System

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Considering a bistable system driven by additive and multiplicative colored noises with colored cross-correlation, we obtain the analytic expressions of the stationary probability distribution $P_{st}(x)$, the linear relaxation time T_c, and the correlated function $C(s)$. The effects of the noise intensity, the self-correlation time and the cross-correlation time for the bistable system are discussed. The noise intensity D speeds up relaxation of the system from unstable points, which when $D < Q$, the effects are the most obvious; when $D > Q$, the effects are damped. The self-correlation time τ_1 and τ_2 make the stationary probability distribution of the dynamical variable x be shaper and speed up the fluctuation decay of the dynamical variable x. On the contrary, the cross-correlation time τ_3 makes the stationary probability distribution of the dynamical variable x be flatter and slows down the fluctuation decay of the dynamical variable x. The effect of the self-correlation time is more projecting than the effect of the cross-correlation time.

KEY WORDS: Bistable system, cross-correlated colored noises, probability distribution, relaxation time, correlation function
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1. INTRODUCTION

Dynamical properties of a bistable system with noises is a very typical and important problem in statistical mechanics. Formerly, in most the previous works, the noises were treated as to be uncorrelated, since it was assumed that they have different noise origins. However, Fedehenia,[(1)] and Fulinski and Telejko[(2)] pointed out that the noises in some stochastic process may have a common origin, and

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thus may be correlated, for example as in laser dynamics the case is shown by Ref. 4. There are also some situations where strong external noises can lead to some changes of the internal structure of a system so that the internal noise and the external noise should be independent.(2,4) Thereafter many physicist have widely started to study the effects with correlations between additive and multiplicative noises in stochastic systems, and found some interesting results.(3,5–13) Hanggi *et al.*(14–16) early begin to investigate "colored noise" issue. Illustrating the Mori theory and the Dupuis algorithm, Faetti and Grigolini *et al.* described time behavior of non-linear stochastic processes in the presence of multiplicative noise: from Kramers' to Suzuki's decay.(17) Using the Novikov theorem and the steady-state value of the deterministic theory, Jia and Li analyzed the steady-state properties of the bistable kinetic model with cross-correlated additive and multiplicative white noises.(5) Applying means of the projection operator method, Xie and Mei investigated dynamical properties of a bistable kinetic model with correlated white noises,(18) and then Mei *et al.*(19,20) investigated the effects of cross-correlation for the relaxation time and the correlated function of a bistable system and shown the dynamical properties of a bistable system with cross-correlated white noises. In 1994, Gammaitoni *et al.*(21) discussed stochastic resonance with interplay of additive and multiplicative noises. Recently, as the ongoing studying works further deepen, people have been more and more interested in the stochastic system with self-correlated and cross-correlated additive and multiplicative colored noises. Assuming the self-correlation time and the cross-correlation time of the additive and the multiplicative noise to be the same value, Ling *et al.*(10) discussed moments of intensity of single-mode laser driven by additive and multiplicative colored noises with colored cross-correlation. Considering the self-correlation time of the multiplicative noise $\tau_1 \neq 0$, the self-correlation time of the additive noise $\tau_2 = 0$, and the cross-correlation time $\tau_3 = 0$, Ling *et al.*(22) yet discussed the stationary probability distribution of a symmetric bistable system with cross-correlation additive and multiplicative noises. In 2005, Borromeo *et al.*(23) shown that a finite additive and multiplicative noise correlation can induce spatial asymmetries in a bistable system. From these researches, we see that the effects of the self-correlation time and the cross-correlation time between additive and multiplicative noises for a bistable system is very interesting.

The purpose of this paper is to discuss the effects of the self-correlation time and the cross-correlation time between additive and multiplicative colored noises for a bistable system. In Sec. 2, we introduce the approximative Fokker–Planck equation (AFPE) for a bistable system with self-correlated and cross-correlated additive and multiplicative colored noises, and solve the AFPE for stationary probability distribution (SPD) of the system. By using the means of the projection operator method, in which the effects of the memory kernels are taken into account, the analytic expressions of the normalized correlation function and the relaxation time on a bistable system were derived. In Sec. 3, based on the numerical results, we discuss the stationary probability distribution and the effects of the noise intensity, the self-correlation time and cross-correlation time for the bistable system, so that we show further the important effects of cross-correlated colored noises to the dynamical properties of a bistable system.

2. STATIONARY PROBABILITY DISTRIBUTION, RELAXATION TIME, AND CORRELATION FUNCTION

We consider a conventional symmetric bistable kinetic system driven by cross-correlated additive and multiplicative colored noises, in which characteristic of the cross-correlation time and self-correlation time of the noises may be different. The Langevin equation of this general system reads

$$\frac{dx}{dt} = x - x^3 + x\xi(t) + \eta(t). \tag{1}$$

Here $\xi(t)$ and $\eta(t)$ are zero-mean Gaussia noises, whose statistical properties are

$$\langle \xi(t) \rangle = \langle \eta(t) \rangle = 0, \tag{2}$$

$$\langle \xi(t)\xi(t') \rangle = \frac{D}{\tau_1} \exp\left(-\frac{|t-t'|}{\tau_1}\right), \tag{3}$$

$$\langle \eta(t)\eta(t') \rangle = \frac{Q}{\tau_2} \exp\left(-\frac{|t-t'|}{\tau_2}\right), \tag{4}$$

$$\langle \xi(t)\eta(t') \rangle = \langle \eta(t)\xi(t') \rangle = \frac{\lambda\sqrt{DQ}}{\tau_3} \exp\left(-\frac{|t-t'|}{\tau_3}\right), \tag{5}$$

where D and Q are the multiplicative colored noise and the additive colored noise intensity, respectively. τ_1 and τ_2 are the self-correlate times of the multiplicative noise and the additive noise, and τ_3 is the cross-correlation time between the multiplicative and the additive colored noise. By virtue of the Novikov theorem,[30] Fox's approach[31] and the ansatz of Hanggi *et al.*,[32] the approximate Fokker–Planck equation corresponding to Eqs. (1) with (2)–(5) is obtained by Refs. 5,12 and 14.

$$\frac{\partial P(x,t)}{\partial t} = L_{\mathrm{FP}} P(x,t), \tag{6}$$

$$L_{\mathrm{FP}} = -\frac{\partial}{\partial x} f(x) + \frac{\partial^2}{\partial x^2} G(x), \tag{7}$$

where

$$f(x) = x - x^3 + \frac{Dx}{1+2\tau_1} + \frac{\lambda\sqrt{DQ}}{1+2\tau_3}, \tag{8}$$

and

$$G(x) = \frac{Dx^2}{1+2\tau_1} + \frac{2\lambda\sqrt{DQ}x}{1+2\tau_3} + \frac{Q}{1+2\tau_2}. \tag{9}$$

Note that the approximate Fokker–Planck equation (6) holds under the conditions of $1+2\tau_1 > 0$, $1+2\tau_2 > 0$, and $1+2\tau_3 > 0$, so there is no restriction on τ_1, τ_2, and τ_3 for $\tau_1 \geq 0$, $\tau_2 \geq 0$, and $\tau_3 \geq 0$.[5] In the case of a stationary state, the probability distribution function $P_{st}(x)$ of Eq. (6) can be obtained below.

1. When the self-correlated times and the cross-correlated time satisfy $\tau_1 = \tau_2 = \tau_3 = \tau$, the stationary probability distribution of the bistable system given by

$$\begin{aligned} P_{st}(x) = N &\left(\frac{Dx^2}{1+2\tau} + \frac{2\lambda\sqrt{DQ}x}{1+2\tau} + \frac{Q}{1+2\tau}\right)^{\beta_1} \\ &\times \exp\left[-\frac{1+2\tau}{2D}x^2 + 2\lambda(1+2\tau)\sqrt{\frac{Q}{D^3}}x\right] \\ &\times \exp\left[\beta_2 \arctan\left(\frac{\sqrt{\frac{D}{Q}}x+\lambda}{\sqrt{1-\lambda^2}}\right)\right] \end{aligned} \tag{10}$$

for $0 \leq |\lambda| < 1$ and

$$\begin{aligned} P_{st}(x) = N &\left(\frac{Dx^2}{1+2\tau} + \frac{2\lambda\sqrt{DQ}x}{1+2\tau} + \frac{Q}{1+2\tau}\right)^{\beta_1} \\ &\times \exp\left[-\frac{1+2\tau}{2D}x^2 + 2\lambda(1+2\tau)\sqrt{\frac{Q}{D^3}}x\right] \\ &\times \exp\left[\frac{-(1+2\tau)}{Dx+\lambda\sqrt{DQ}}\right] \end{aligned} \tag{11}$$

for $|\lambda| = 1$, where

$$\beta_1 = \frac{1+2\tau}{2D}\left[1+\frac{Q}{D}(1-4\lambda^2)\right] - \frac{1}{2},$$

and

$$\beta_2 = -\frac{\lambda(1+2\tau)}{D\sqrt{1-\lambda^2}}\left[1+\frac{Q}{D} - \frac{2Q}{D}(2\lambda^2-1)\right].$$

2. When the self-correlated times and the cross-correlated time satisfy $\tau_1 = \tau_2$ and $\tau_3 > \tau_1$, the stationary probability distribution of the bistable system

given by

$$P_{st}(x) = N\left(\frac{Dx^2}{1+2\tau_1} + \frac{2\lambda\sqrt{DQ}x}{1+2\tau_3} + \frac{Q}{1+2\tau_1}\right)^{\alpha_1} \times \exp\left[-\frac{1+2\tau_1}{2D}x^2 + \frac{2\lambda\sqrt{DQ}(1+2\tau_1)^2}{D^2(1+2\tau_3)}x\right] \times \exp\left[\alpha_2 \arctan\left[\frac{\sqrt{\frac{D}{Q}}\frac{x}{1+2\tau_1} + \frac{\lambda}{1+2\tau_3}}{\sqrt{\frac{1}{(1+2\tau_1)^2} - \frac{\lambda^2}{(1+2\tau_3)^2}}}\right]\right] \tag{12}$$

for $0 \le |\lambda| \le 1$, where

$$\alpha_1 = \frac{1+2\tau_1}{2D}\left[1 + \frac{Q}{D}\left(1 - \frac{4\lambda^2(1+2\tau_1)^2}{(1+2\tau_2)^2}\right)\right] - \frac{1}{2},$$

and

$$\alpha_2 = -\frac{\lambda\left(\frac{1+2\tau_1}{1+2\tau_3}\right)}{D\sqrt{\frac{1}{(1+2\tau_1)^2} - \frac{\lambda^2}{(1+2\tau_3)^2}}} \times\left\{1 + \frac{Q}{D} - \frac{2Q}{D}(1+2\tau_1)^2\left[\frac{2\lambda^2}{(1+2\tau_3)^2} - \frac{1}{(1+2\tau_1)^2}\right]\right\}.$$

3. When the self-correlated times and the cross-correlated time satisfy $\tau_1 = \tau_2$ and $\tau_3 < \tau_1$, the discriminant $\Delta = 4DQ[\lambda^2/(1+2\tau_3)^2 - 1/(1+2\tau_1)^2]$, plus-minus of which is determined by taking values of τ_1, τ_3, and λ. The stationary probability distribution of the bistable system given by

$$P_{st}(x) = N\left(\frac{Dx^2}{1+2\tau_1} + \frac{2\lambda\sqrt{DQ}x}{1+2\tau_3} + \frac{Q}{1+2\tau_1}\right)^{\frac{1+2\tau_1}{2D}[1-\frac{3Q}{D}]} \times \exp\left[-\frac{1+2\tau_1}{2D}x^2 + \frac{2\lambda\sqrt{DQ}(1+2\tau_1)^2}{D^2(1+2\tau_3)}x\right] \times \exp\left[\frac{\lambda\sqrt{DQ}}{D}\left(\frac{1+2\tau_1}{1+2\tau_3}\right)\left(-1+\frac{Q}{D}\right)\frac{-1}{\frac{Dx}{1+2\tau_1} + \frac{\lambda\sqrt{DQ}}{1+2\tau_3}}\right] \tag{13}$$

for $\Delta = 0$,

$$P_{\rm st}(x) = N\left(\frac{Dx^2}{1+2\tau_1} + \frac{2\lambda\sqrt{DQ}x}{1+2\tau_3} + \frac{Q}{1+2\tau_1}\right)^{\alpha_1} \times \exp\left[-\frac{1+2\tau_1}{2D}x^2 + \frac{2\lambda\sqrt{DQ}(1+2\tau_1)^2}{D^2(1+2\tau_3)}x\right] \times \exp\left[\alpha_2 \arctan\left[\frac{\sqrt{\frac{D}{Q}}\frac{x}{1+2\tau_1} + \frac{\lambda}{1+2\tau_3}}{\sqrt{\frac{1}{(1+2\tau_1)^2} - \frac{\lambda^2}{(1+2\tau_3)^2}}}\right]\right] \tag{14}$$

for $\Delta < 0$.

In Eqs. (10)–(14), N is the responding normalization constant. The normalization constant N is given by the equation

$$\int_{-\infty}^{\infty} P_{\rm st}(x)dx = 1.$$

The expectation values of the nth power of the state variable x are given by

$$\langle x^n \rangle = \int_{-\infty}^{\infty} x^n P_{\rm st}(x)dx. \tag{15}$$

For a general stochastic process which a stationary state exists, the stationary correlation function is defined by

$$C(s) = \langle \delta x(t+s)\delta x(t)\rangle_{\rm st} = \lim_{t\to\infty} \langle \delta x(t+s)\delta x(t)\rangle, \tag{16}$$

where

$$\delta x(t) = x(t) - \langle x(t)\rangle.$$

A normalized correlation function is

$$C(s) = \frac{\langle \delta x(t+s)\delta x(t)\rangle_{\rm st}}{\langle (\delta x)^2\rangle_{\rm st}}. \tag{17}$$

The associated relaxation time which describes the fluctuation decay of the dynamical variable x is defined by

$$T_c = \int_0^{\infty} C(t)dt. \tag{18}$$

By using the projection operator method,[19] the zeroth approximation for the relaxation time is given by

$$T_c = \gamma_0^{-1} = \frac{\langle (\delta x)^2\rangle_{\rm st}}{\langle G(x)\rangle_{\rm st}}. \tag{19}$$

Similarly, the first-order approximation for the relaxation time is given by

$$T_c = \left[\gamma_0 + \frac{\eta_1}{\gamma_1}\right]^{-1}, \tag{20}$$

where

$$\eta_1 = \frac{\langle G(x) f'(x)\rangle_{st}}{\langle(\delta x)^2\rangle_{st}} + \gamma_0^2, \tag{21}$$

and

$$\gamma_1 = -\frac{\langle G(x)[f'(x)]^2\rangle_{st}}{\eta_1\langle(\delta x)^2\rangle_{st}} + \frac{\gamma_0^3}{\eta_1} - 2\gamma_0. \tag{22}$$

Employing Eqs. (8), (9), and (15), we have

$$\gamma_0 = \frac{k_2}{\langle(x)^2\rangle_{st} - \langle x\rangle_{st}^2}, \tag{23}$$

$$\eta_1 = \frac{\left(1 + \frac{D}{1+2\tau_1}\right)k_2 - 3k_4}{\langle(x)^2\rangle_{st} - \langle x\rangle_{st}^2} + \gamma_0^2, \tag{24}$$

and

$$\gamma_1 = \frac{-1}{\eta_1(\langle(x)^2\rangle_{st} - \langle x\rangle_{st}^2)}\left[\left(1 + \frac{D}{1+2\tau_1}\right)^2 k_2 - 6 \times\left(1 + \frac{D}{1+2\tau_1}\right)k_4 + 9k_6\right] + \frac{\gamma_0^3}{\eta_1} - 2\gamma_0, \tag{25}$$

where

$$k_2 = \frac{D}{1+2\tau_1}\langle x^2\rangle_{st} + \frac{2\lambda\sqrt{DQ}}{1+2\tau_3}\langle x\rangle_{st} + \frac{Q}{1+2\tau_2}, \tag{26}$$

$$k_4 = \frac{D}{1+2\tau_1}\langle x^4\rangle_{st} + \frac{2\lambda\sqrt{DQ}}{1+2\tau_3}\langle x^3\rangle_{st} + \frac{Q}{1+2\tau_2}\langle x^2\rangle_{st}, \tag{27}$$

and

$$k_6 = \frac{D}{1+2\tau_1}\langle x^6\rangle_{st} + \frac{2\lambda\sqrt{DQ}}{1+2\tau_3}\langle x^5\rangle_{st} + \frac{Q}{1+2\tau_2}\langle x^4\rangle_{st}. \tag{28}$$

Here, we see that the zeroth approximation of the relaxation time $T_c = \gamma_0^{-1}$ is good agreement with the result calculated by virtue of the Stratonovich-like ansatz.[33] When $\lambda = 0$ and $Q = 0$, the above results fall back to Eqs. (2.29)–(2.31), as presented in Ref. 34. In other words, the Stratonovich-like ansatz completely neglects the memory kernel.

Applying the projection operator method and performing the Laplace transformation,[19] we easily get the stationary normalized correlation function of the variable x

$$C(s) = \beta \exp(-\pi_- s) + (1 - \beta) \exp(-\pi_+ s), \tag{29}$$

where

$$\beta = \frac{\gamma_1 - \pi_-}{\pi_+ - \pi_-} \tag{30}$$

and

$$\pi_\pm = \frac{\gamma_0 + \gamma_1}{2} \pm \frac{1}{2}\sqrt{(\gamma_0 - \gamma_1)^2 - 4\eta_1}. \tag{31}$$

3. DISCUSSION AND CONCLUSION

Using the expression Eqs. (10)–(14) of the stationary probability distribution (SPD) of the bistable system with cross-correlated additive and multiplicative colored noises, the effects of the cross-correlation strength λ, the self-correlation time, the cross-correlation time, and the noise intensity can be studied conveniently. By numerical computation, we plotted the curves of the SPD of the bistable system in Figs. 1–3.

When the self-correlation times and the cross-correlation time satisfying $\tau_1 = \tau_2 = \tau_3 = \tau$, the SPD of the system versus the variable x is plotted in Fig. 1. In Fig. 1a, τ is fixed to be 0.2 and λ takes different values. In Fig. 1b, λ is fixed to be 0.2 and τ takes different values. From Fig. 1, we see that the SPD exhibits two peaks the posits of which distribute symmetrically at the left and the right of the origin, respectively. When τ is fixed, for $\lambda > 0$, the λ enhances the left peak of SPD of the bistable system and attenuates the right peak; on the contrary, for $\lambda < 0$, the $|\lambda|$ enhances the right peak of the SPD of the bistable system and attenuates the left peak ;The larger $|\lambda|$ is, the more projecting the effect of $|\lambda|$ is. When λ is fixed, for $\lambda > 0$, τ enhances the left peak of the SPD of the bistable system and attenuates the right peak. For $\lambda < 0$, τ enhances the right peak of the SPD of the bistable system and attenuates the left peak; The larger τ is, the more projecting the effect of τ is.

In Fig. 2, the self-correlation times satisfies $\tau_1 = \tau_2$, $\tau_3 > \tau_1$, and $\lambda = \pm 0.8$. τ_3 is fixed to be 4.1 for Fig. 2a and τ_1 is fixed to be 0.5 for Fig. 2b. When the self-correlation time satisfying $\tau_1 = \tau_2$, $\tau_3 < \tau_1$, the discriminant $\Delta = 4DQ[\lambda^2/(1 + 2\tau_3)^2 - 1/(1 + 2\tau_1)^2]$, plus-minus of which is determined by taking values of τ_1, τ_3, and λ. In Fig. 3a, $\lambda = \pm 0.8$, $\tau_1 = 2.0$, $1.5 < \tau_3 < 2.0$, and $\Delta < 0$. From Fig. 2a, we see that for $\lambda > 0$, τ_1 enhances the left peak of the SPD of the bistable system and attenuates the right peak; for $\lambda < 0$, τ_1 enhances the right peak of the SPD of the bistable system and attenuates the left peak. From Figs. 2b and 3a, we

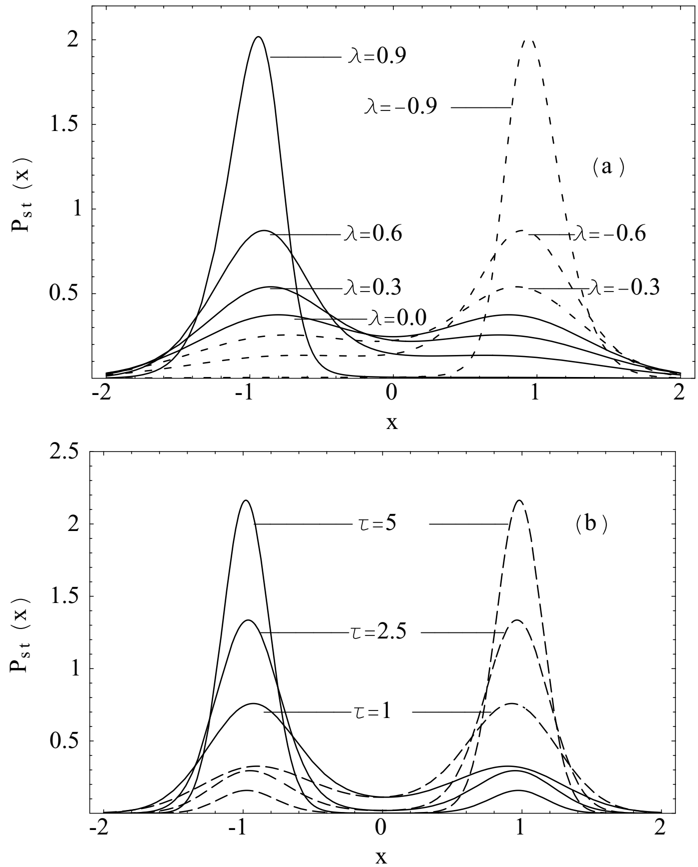


Fig. 1. The stationary probability distribution $P_{st}(x)$ vs the variable x for $D = 0.5$, $Q = 0.25$ and $\tau_1 = \tau_2 = \tau_3 = \tau$. (a) $\tau = 0.2$, λ takes 0, ± 0.3, ± 0.6, and ± 0.9, respectively. (b) $\lambda = 0.8$ (*solid line*), $\lambda = -0.8$ (*dashed line*), and τ takes 1, 2.5, and 5, respectively.

see that for $\lambda > 0$, τ_3 attenuates the left peak of the SPD of the bistable system and enhances the right peak; for $\lambda < 0$, τ_3 attenuates the right peak of the SPD of the bistable system and enhances the left peak.

Equations (11) and (13) show that when $x = -(1 + 2\tau_1)\lambda\sqrt{Q/D}/(1 + 2\tau_3)$, the SPD exhibits divergence. In Fig. 3b and c, $|\lambda| = 0.60$, $\tau_1 = 2$, $\tau_3 = 1$, $\Delta = 0$, the SPD diverges at $x = \pm 0.707$. $\lambda > 0$ is for Fig. 3b and $\lambda < 0$ is for Fig. 3c.

The correlated function $C(s)$ describes the dynamical fluctuation decay of the variable x with time in the stationary state. The normalized correlated functions $C(s)$ of the bistable system versus the decay time s are shown in Fig. 4. Obviously, $C(s)$ decreases exponentially as the decay time s increases. In Fig. 3a, $\tau_3 = 4.1$,

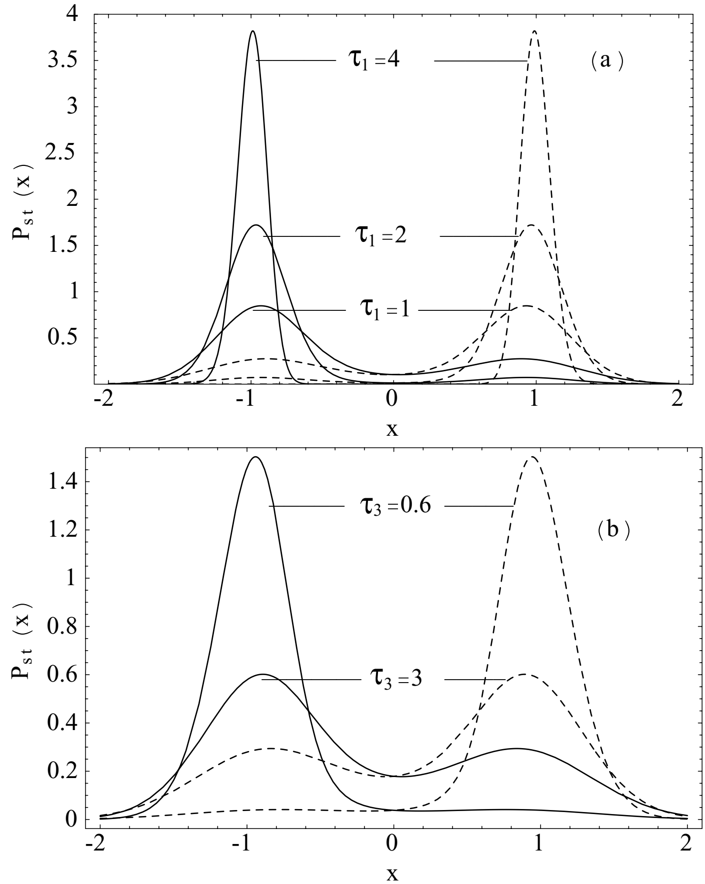


Fig. 2. The stationary probability distribution $P_{st}(x)$ vs the variable x for $D = 0.5$, $Q = 0.25$ and $\tau_1 = \tau_2$. (a) *Solid line* for $\lambda = 0.8$, *dashed line* for $\lambda = -0.8$, $\tau_3 = 4.1$, τ_1 takes 1, 2, and 4, respectively. (b) *Solid line* for $\lambda = 0.8$, *dashed line* for $\lambda = -0.8$, $\tau_1 = 0.5$, and τ_3 takes 0.6 and 3, respectively.

$|\lambda| = 0.80$, and τ_1 takes different values. The larger τ_1 is, the smaller the value of $C(s)$ is. τ_1 attenuates the dependency between the state variables of the bistable system at different times. In Fig. 3b, $\tau_1 = 0.5$, $|\lambda| = 0.80$, and τ_3 takes different values. The larger τ_3 is, the larger the value of $C(s)$ is. τ_3 enhances the dependency between the state variables of the bistable system at different times .

The relaxation time gives dynamical information about the time scale of the evolution of a spontaneous fluctuation in the steady state and reflects the evolution velocity of the system from a arbitrary initial state to the stationary state.[34–37] The linear relaxation time T_c versus the multiplicative noise intensity D is plotted in

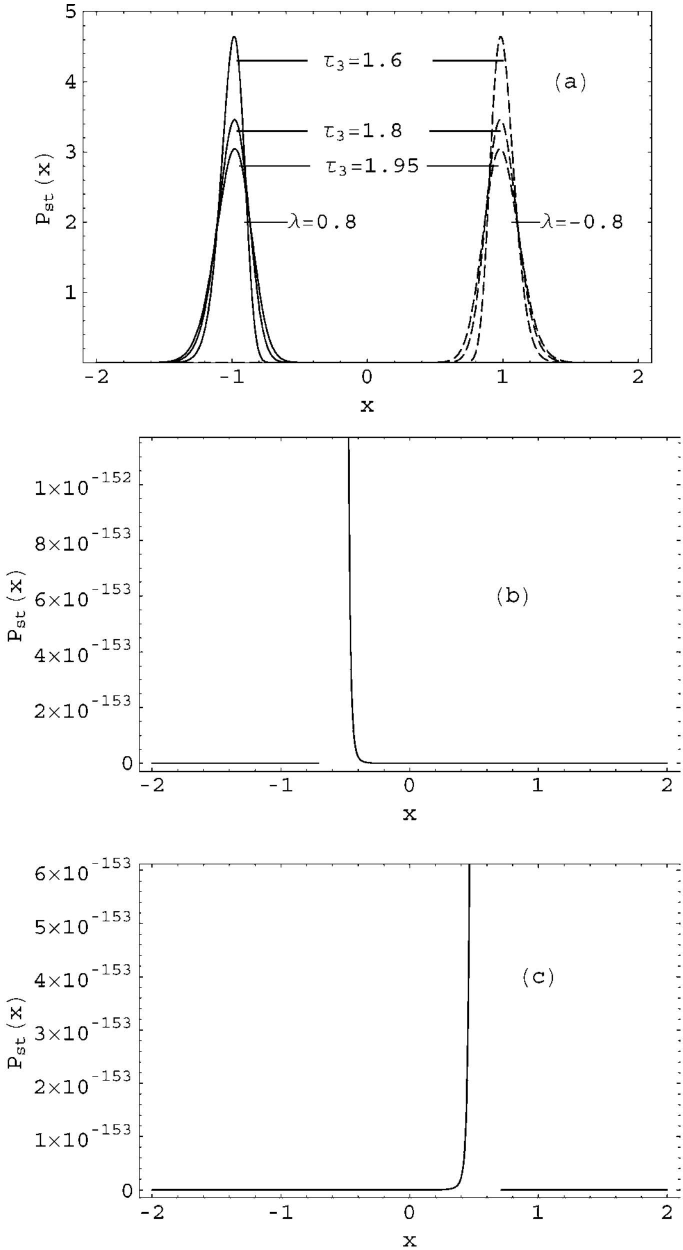


Fig. 3. The stationary probability distribution $P_{\mathrm{st}}(x)$ vs the variable x for $D = 0.5$, $Q = 0.25$ and $\tau_1 = \tau_2$. (a) $\tau_1 = 2$, $\lambda = 0.8$ (*solid line*), $\lambda = -0.8$ (*dashed line*), and τ_3 takes 1.6, 1.8, and 1.95, respectively. (b) $\tau_1 = 2$, $\tau_3 = 1$, and $\lambda = 0.6$. (c) $\tau_1 = 2$, $\tau_3 = 1$, and $\lambda = -0.6$.

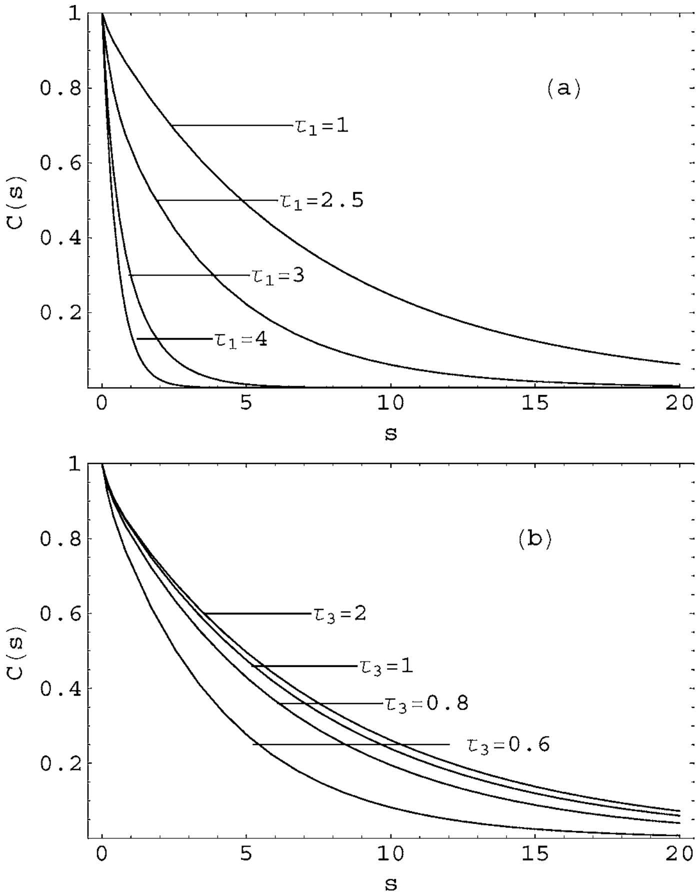
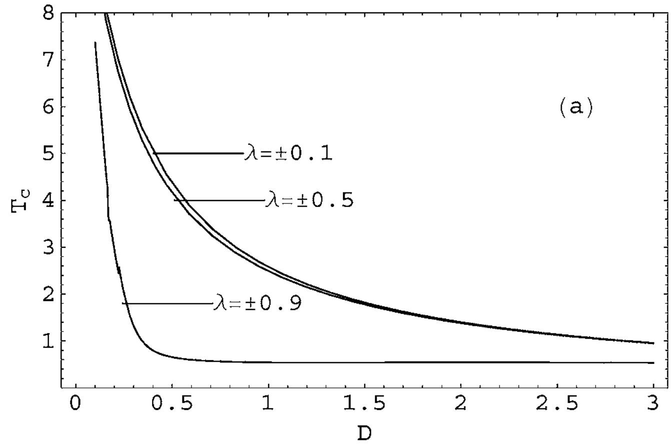


Fig. 4. The normalized correlation function $C(s)$ as a function of decay time s for $D = 1$, $Q = 0.25$, $|\lambda| = 0.8$, and $\tau_1 = \tau_2$. (a) $\tau_3 = 4.1$. τ_1 takes 1, 2.5, 3, and 4, respectively. (b) $\tau_1 = 0.5$. τ_3 takes 0.6, 0.8, 1, and 2, respectively.

Fig. 5 ($\tau_1 = \tau_2 = \tau_3 = \tau$). From Fig. 5, we see that the relaxation time T_c decreases monotonously as the noise intensity D increases, D speeds up the relaxation of the system from unstable points, which when $D < Q$, the effects are most obvious; when $D > Q$, the effects are damped. In Fig. 5a, the cross-correlation time is fixed to be 0.5, the relaxation time T_c decreases as the cross-correlation strength $|\lambda|$ increases. In Fig. 5b, the cross-correlation strength $|\lambda|$ is fixed to be 0.5, the relaxation time T_c increases as the cross-correlation time τ increases.

From above discussion, we understand that cross-correlated additive and multiplicative colored noises play an important role in a bistable system. In the bistable



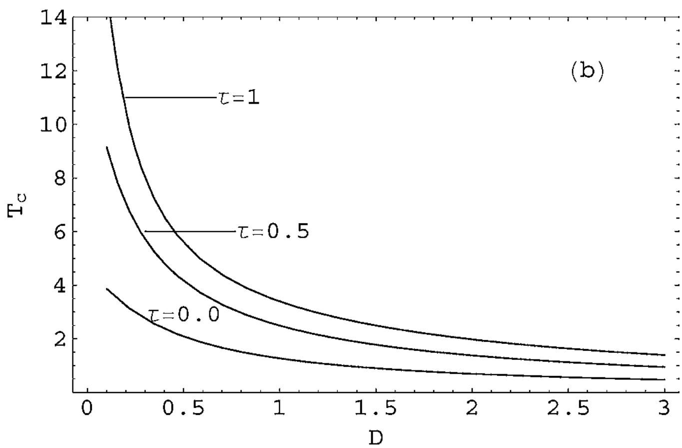


Fig. 5. The linear relaxation time T_c as a function of the noise intensity D for $\tau_1 = \tau_2 = \tau_3 = \tau$, and $Q = 0.25$. (a) $\tau = 0.5$, λ takes ± 0.1, ± 0.5, and ± 0.9, respectively. (b) $\lambda = \pm 0.5$, τ takes 0, 0.5, and 1, respectively.

system with cross-correlated additive and multiplicative colored noises, for $\lambda > 0$, λ enhances the stationary probability distribution of the dynamical variable $x < 0$ and attenuates the stationary probability distribution the dynamical variable $x > 0$; on the contrary, for $\lambda < 0$, $|\lambda|$ enhances the stationary probability distribution of the dynamical variable $x > 0$ and attenuates the stationary probability distribution of the dynamical variable $x < 0$. The noise intensity $D(Q$ is fixed to be 0.25) and the cross-correlation strength $|\lambda|$ speed up the system relaxation from unstable points, but the cross-correlation time slows down the system relaxation from unstable points. The self-correlation times τ_1 and τ_2 make the stationary probability distribution of the dynamical variable x be shaper and speed up the fluctuation decay of the of the dynamical variable x. On the contrary, the cross-correlation time τ_3 makes the stationary probability distribution of the dynamical variable x be flatter and slow down the fluctuation decay of of the dynamical variable x. In Fig. 1b, $\tau_1 = \tau_2 = \tau_3 = \tau$, which the action magnitude of the self-correlation time is same with the action magnitude of the cross-correlation time, but τ enhances still the fluctuation of the SPD, that is to say, the effect of the self-correlation time is more projecting than the effect of the cross-correlation time.

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